Chapter 3

Perpendicular and Parallel Lines

Section 4
Proving Lines are Parallel

GOAL 1: Proving Lines are Parallel

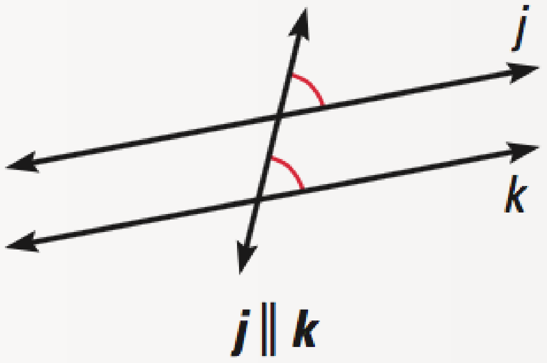
To use the theorems you learned in Lesson 3.3, you must first know that two lines are parallel. You can use the following postulate and theorems to prove that two lines are parallel.

Converse of Postulate 15

POSTULATE

POSTULATE 16 ***Corresponding Angles Converse***

If two lines are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.

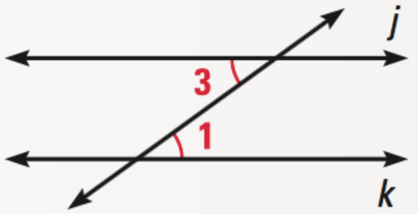


The following theorems are converses of those 3.4 in Lesson 3.3. Remember that the converse of a true conditional statement is not necessarily true. 3.5 Thus, each of the following must be proved to be true. Theorems 3.8 and 3.9 are proved in Examples 3.6 1 and 2. You are asked to prove Theorem 3.10 in Exercise 30.

THEOREMS ABOUT TRANSVERSALS

THEOREM 3.8 ***Alternate Interior Angles Converse***

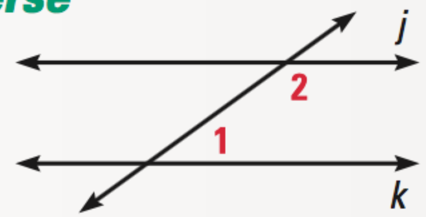
If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel.



If $\angle 1 \cong \angle 3$, then $j \parallel k$.

THEOREM 3.9 ***Consecutive Interior Angles Converse***

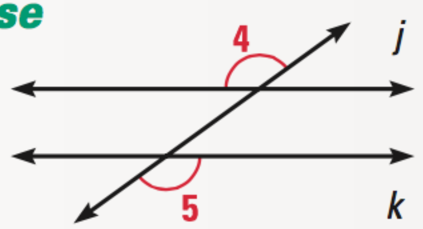
If two lines are cut by a transversal so that consecutive interior angles are supplementary, then the lines are parallel.



If $m\angle 1 + m\angle 2 = 180°$, then $j \parallel k$.

THEOREM 3.10 ***Alternate Exterior Angles Converse***

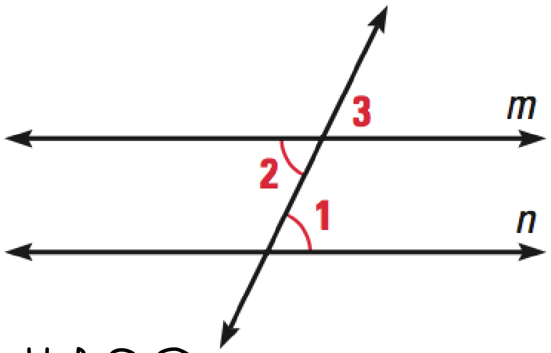
If two lines are cut by a transversal so that alternate exterior angles are congruent, then the lines are parallel.



If $\angle 4 \cong \angle 5$, then $j \parallel k$.

Example 1: Proof of the Alternate Interior Angles Converse

Prove the Alternate Interior Angles Converse.



Given

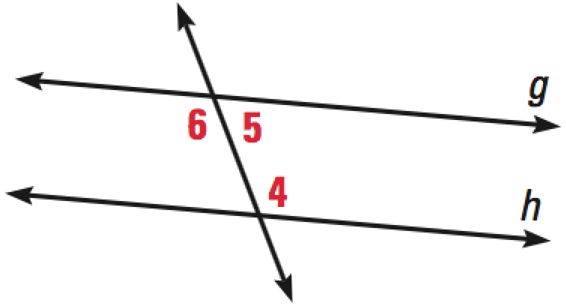
If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel.

prove

S	R
<1 cong. <2	Given
<2 cong. <3	Vertical Angles Theorem
<1 cong. <3	Transitive
***corresponding <‘s	
m || n	Corresponding <‘s CONVERSE

When you prove a theorem you may use only earlier results. For example, to prove Theorem 3.9, you may use Theorem 3.8 and Postulate 16, but you may not use Theorem 3.9 itself or Theorem 3.10.

Example 2: Proof of the Consecutive Interior Angles Converse



Prove the Consecutive Interior Angles Converse.

Given

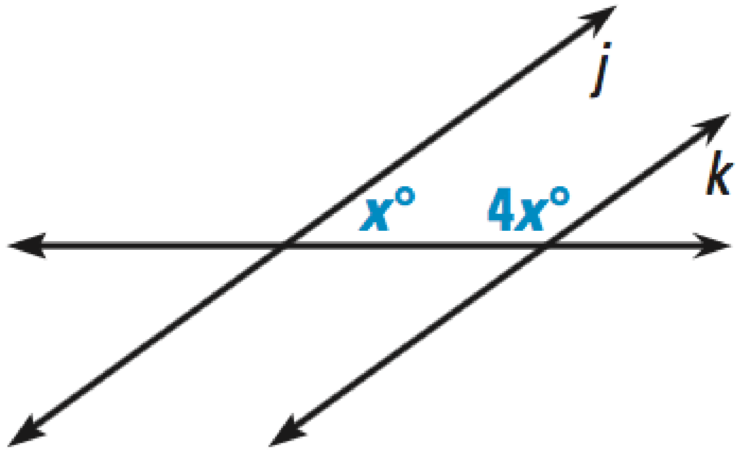
If two lines are cut by a transversal so that consecutive interior angles are supplementary, then the lines are parallel.

prove

S	R
<4 & <5 are supplementary	Given
<5 & <6 are supplementary	Linear Pair Postulate
<4 cong. <6	Cong. Supplements Theorem (2.4)
g || h	Alt. Int. <'s CONVERSE

Example 3: Applying the Consecutive Interior Angles Converse

Find the value of x that makes *j* || *k*.



Same-side interior → supplementary

$$x + 4x = 180$$

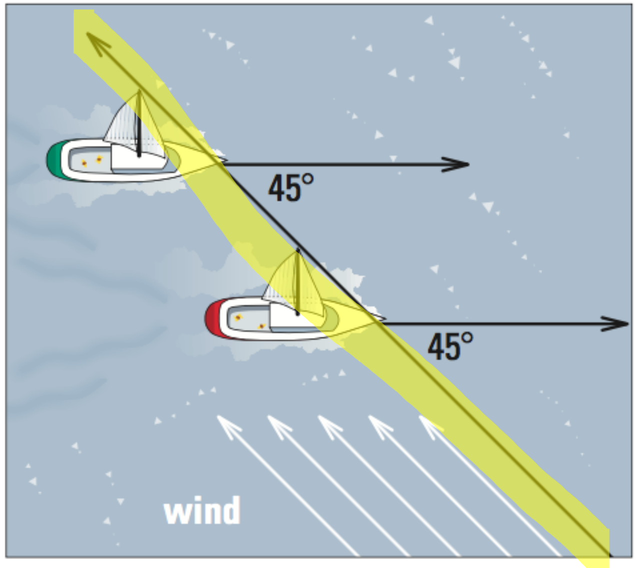
$$\frac{5x}{5} = \frac{180}{5}$$

$$x = 36$$

GOAL 2: Using the Parallel Converses

Example 4: Using the Corresponding Angles Converse

SAILING If two boats sail at a 45° angle to the wind as shown, and the wind is constant, will their paths ever cross? Explain.



B/c the corresponding <‘s are congruent

→ paths (lines) are parallel

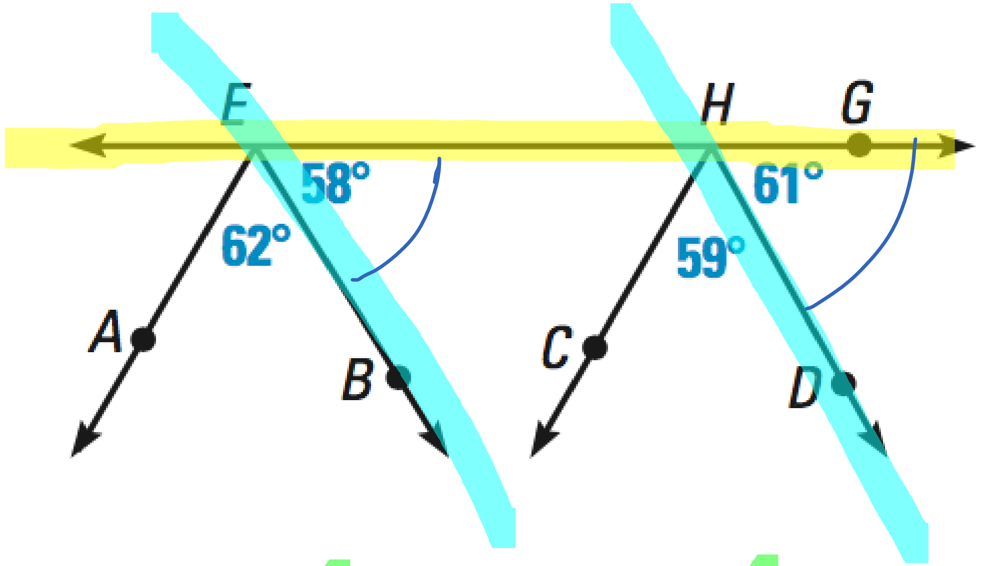
→ won’t cross

Example 5: Identifying Parallel Lines

Decide which rays are parallel.

a. Is $\overrightarrow{EB}$ parallel to $\overrightarrow{HD}$?

Corr. <‘s aren’t congruent → not parallel



a. Is $\overrightarrow{EA}$ parallel to $\overrightarrow{HC}$?

<AEH → 62 + 58 = 120*

<CHG → 59 + 61 = 120*

Corr. <‘s are congruent → are parallel